

LOGICA

COMPITO 5

(A)

$$1) \vdash \neg \forall x \neg \forall y Ryx \rightarrow \neg \exists y \neg \exists x Ryx$$

$$1 \neg (\neg \forall x \neg \forall y Ryx \rightarrow \neg \exists y \neg \exists x Ryx)$$

$$2 \neg \forall x \neg \forall y Ryx$$

$$3 \neg \exists y \neg \exists x Ryx$$

$$3a \exists y \neg \exists x Ryx$$

$$4 \exists x \neg \forall y Ryx$$

$$5 \forall y Rya$$

$$6 \neg \exists x Rbx$$

$$7 \forall x \neg Rbx$$

$$8 \neg Rba$$

$$9 Rba$$

$$10 \times 8, 9 \neg$$

È UNA TAUTOLOGIA.

$$2) \vdash \neg \forall x \forall y (x=y \leftrightarrow y=x)$$

$$1 \neg \neg \forall x \forall y (x=y \leftrightarrow y=x)$$

$$2 \forall x \forall y (x=y \leftrightarrow y=x)$$

$$3 \forall y (a=y \leftrightarrow y=a)$$

$$4 a=b \leftrightarrow b=a$$

$$5 \begin{array}{l} a=b \\ b=a \end{array} \quad \begin{array}{l} \neg a=b \\ \neg b=a \end{array}$$

È INVALIDA.

1, $\neg \rightarrow$

1, $\neg \rightarrow$

3, $\neg \neg$

2, $\neg \forall$

4, $\exists$

3, $\exists$

6, $\neg \exists$

7, $\forall$

9, $\forall$

1, $\neg \neg$

2, $\forall$

3, $\forall$

4, $\leftrightarrow$

B

②.

$$1 \exists x(Fx \& \exists y Gy) \wedge \vdash \exists x \exists y (Fx \& Gy)$$

$$2 \mid Fa \& \exists y Gy \quad \text{H pr E}$$

$$3 \mid Fa \quad \text{E \& 2}$$

$$4 \mid \exists y Gy \quad \text{E \& 2}$$

$$5 \mid \mid Gb \quad \text{H pr E}$$

$$6 \mid Fa \& Gb \quad \text{1 \& 3-5}$$

$$7 \mid \exists x(Fx \& Gb) \quad \text{1} \exists 6$$

$$8 \mid \exists x \exists y (Fx \& Gy) \quad \text{1} \exists 7$$

$$9 \mid \exists x \exists y (Fx \& Gy) \quad \text{4, 5-9} \exists E$$

$$10 \exists x \exists y (Fx \& Gy) \quad \text{1, 2-9} \exists E$$

③.

$$1 \forall x(Fx \rightarrow Gx) \wedge \vdash \sim Gb \rightarrow \sim b=a$$

$$2 \mid Fa \quad \text{A}$$

$$3 \mid Fa \rightarrow Ga \quad \text{E} \forall 1$$

$$4 \mid Ga$$

$$5 \mid \sim Gb \quad \text{HI} \rightarrow$$

$$6 \mid \mid b=a \quad \text{HI} \sim$$

$$7 \mid \mid Gb \quad \text{E} = 4, 6$$

$$8 \mid \mid \sim Gb \& Gb \quad \text{1 \& 5, 7}$$

$$9 \mid \sim b=a \quad \text{6-8, } \sim I$$

$$10 \sim Gb \rightarrow \sim b=a \quad \text{5-9, } \rightarrow I$$

①.

$$1 \sim \exists x Fx \quad \text{A} \quad \vdash \forall x(Fx \rightarrow Gx)$$

$$2 \mid \sim Ga \quad \text{H pr 1} \rightarrow$$

$$3 \mid \forall x \sim Fx \quad \text{S.Q. 1}$$

$$4 \mid \sim Fa \quad \text{E} \forall 3$$

$$5 \mid \sim Ga \rightarrow \sim Fa$$

$$6 \mid Fa \rightarrow Ga \quad \text{5, TRAS}$$

$$7 \mid \forall x(Fx \rightarrow Gx) \quad \text{1} \forall 6$$