

A RULE FOR AGGREGATING INDIVIDUAL WELFARE MEASURES

Alberto Pench

Department of Economics, University of Pisa, Pisa, Italy

ABSTRACT

It is common, but often incorrect, practice in most applied works, especially in the field of applied general equilibrium models, to evaluate the overall impact of alternative policy changes by means of the sum of equivalent variations of (different classes of) individual consumers. This note proposes to use these same equivalent variations in a different way, namely, to aggregate them by the Borda rule.

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I. INTRODUCTION

The present note originates from reading a paper by Blackorby and Donaldson (1985) and I am indebted to Professor Blackorby for putting it to my attention. Their paper deals with the possibility to make social welfare judgements from the knowledge of consumers' surpluses, where the last terms refer both to Hicksian and to Marshallian measures of individual welfare change. More specifically, Blackorby and Donaldson (1985) assume a given evaluation rule for a cost-benefit analyst based on consumers' surpluses and find conditions for the existence of a social welfare function providing a ranking of alternatives consistent with the ranking of the cost-benefit analyst.

Their results are quite different for Hicksian measures and Marshallian consumer surplus: in the former case, prices must be the same for each

consumer, preferences must be quasi-homothetic and must satisfy the Gorman (1953) conditions for aggregation, and both the social welfare function and the cost–benefit evaluation rule must be linear; in the latter case, prices may differ among consumers, preferences must be homothetic but not identical, the cost–benefit rule must be linear and the social welfare function must be a Cobb–Douglas function. As noted by the authors,¹ their results also apply in a different context: if the cost–benefit rule is Pareto-inclusive, and continuous and social outcomes can be represented by a real-valued function, their paper demonstrates the conditions for the cost–benefit rule to give an ordering, reflexive, complete and transitive, for all possible social outcomes.

An apparently striking feature emerges using Hicksian measures, which require no *a priori* conditions for their use, in contrast to Marshallian consumer surplus: even if the cost–benefit rule and the social evaluation function are the same (i.e., linear), their compatibility is not assured unless the above stringent conditions are fulfilled. The present paper shows how, in the context of Hicksian measures, these restrictions can be overcome by employing the Borda rule both as the cost–benefit rule and as the social evaluation function. Obviously, employing Marshallian consumer surplus requires the well-known conditions for its path independence. As a consequence, and in contrast to some conclusions ‘rehabilitating’ Marshallian consumer surplus in Blackorby and Donaldson (1985, p. 260), the present note gives a strong recommendation in favour of Hicksian measures.

II. BORDA RULE AND AGGREGATION

The idea to start from known consumers’ surpluses of the Blackorby and Donaldson paper is particularly familiar to the writer, given his past experience in the field of applied general equilibrium. It is standard practice in such models to perform simulations of alternative policies (frequently non-marginal tax reforms) and compute their welfare effects in terms of equivalent variations: finally, their sum is often taken as an aggregate impact of the policies under examination.² In this context, it may be interesting to see whether the Blackorby and Donaldson results apply: most of the applied general equilibrium models employ ‘convenient’ functional forms to model the utility functions of the consumers, and a large part of them build a nested structure of linearly homogeneous constant elasticity of substitution functions with differing parameters from consumer to consumer and, possibly, different prices for leisure if a variable labour supply and income taxes are modelled.

¹ Blackorby and Donaldson (1985, p. 252).

² See, however, the critical considerations in Shoven and Whalley (1984, p. 1022).

As a consequence, the conditions of the Blackorby and Donaldson paper are not generally fulfilled.

The link between social welfare functions and the cost–benefit rule in the Blackorby and Donaldson paper suggests the proposal of the present paper, namely, to employ the Borda rule both as a means of aggregating individual surpluses to rank alternative social outcomes and as the social evaluation function.

More precisely, suppose N alternatives and H consumers, whose equivalent variation for each alternative, with respect to a given status quo, is known. According to them, the scores $(N, N - 1, \dots, 1)$, implied by the Borda rule, are imputed, giving the individual ranking of the alternatives. Finally, the sum, over H , of the scores of each alternative gives the cost–benefit evaluation of the alternatives against a status quo. Equivalent variation is an exact and consistent measure to evaluate individual welfare changes with respect to the status quo given by the initial price vector: here, the term exact refers to path independence, whereas the term consistent indicates that equivalent variation is positive (negative) if and only if a proposed alternative is ranked above (under) the status quo in the individual preference ordering represented by the utility function. Indicating with EV^h the equivalent variation of consumer h for a given alternative, this means that

$$EV^h \geq 0 \Leftrightarrow U^h(1) \geq U^h(0)$$

where indices 1 and 0 refer, respectively, to the given alternative and the status quo. Using the Borda rule, both as the cost–benefit rule and as the social evaluation function, means that the sign of the equivalent variations and the difference in utility levels, respectively, are converted into the same score, thus assuring their mutual compatibility without any additional condition.

The current proposal, on the one hand, integrates the results of Blackorby and Donaldson because the evaluation rule itself is a valid, non-continuous, social choice rule, producing an ordering of all possible alternatives without requiring any restriction on preferences. On the other hand, it stresses a result that is implicit in the enormous literature on collective choice in which the richer structure of individual utility functions, with respect to individual preference orderings, has been employed in natural conjunction with social welfare functions to produce various well-known results, often requiring stringent requisites on utility functions in terms of measurability and comparability and on admissible social welfare functions. If it is self-evident, from a theoretical point of view, that the Borda rule admits a representation of individual preferences by means of utility functions, then it is also true, to my knowledge, that such an opportunity has never been seized from an applied perspective to produce an evaluation rule whose informational basis is ordinal non-comparable utility functions.

The Borda rule, in turn, has some well-known shortcomings. Within the framework of Arrow's impossibility theorem, it violates the axiom of independence of irrelevant alternatives, and one of the consequences of this is that social choice has to be based on the whole set of alternatives. This does not seem to be a concern in the present context because this is a case of what Sen (1986, p. 1152) calls the narrow Borda rule in which the scores are based not on the whole set of social alternatives but only on the subset of alternatives over which a choice has to be made.

A related shortcoming is the manipulability of the Borda rule. On the one hand, agents may have an incentive to misrepresent their preferences if they believe that all other agents 'vote' sincerely; on the other hand, the final result may be manipulated by the introduction of further alternatives in a way resembling the power of the agenda setter under majority voting. Neither of the above negative consequences seems to be applicable in the present context: consumers are not requested to put a score on each alternative because the scores are imputed on the basis of equivalent variations, which are (possibly) computed from their past behaviour in the market. The incentive to manipulate the final result by the introduction of further alternatives may be ruled out by separating the responsibilities of defining the alternatives from those of computing equivalent variations.

It may be interesting to close the paper with a simple example showing how such a procedure works and the possible different outcomes with respect to two other rules for aggregating equivalent variations, namely, majority voting and the abovementioned, and commonly used, (unweighted) sum. Let us consider Table 1 in which are shown the equivalent variations of three agents (A, B and C) computed for five alternatives (I, II, III, IV and V) with respect to a given status quo. At the bottom line, labelled by Σ , is computed their sum so that alternative V emerges as the preferred one under this rule.

TABLE 1

	I	II	III	IV	V
A	-2	10	0	3	9
B	4	-10	0	10	8
C	8	10	7	9	6
Σ	10	10	7	22	23

Table 2 is based on the data of the previous reports and the Borda scores and their sum. As can be seen, the preferred alternative is now IV.

TABLE 2

	I	II	III	IV	V
A	1	5	2	3	4
B	3	1	2	5	4
C	3	5	2	4	1
Σ	7	11	6	12	9

Finally, on the basis of the same data, majority voting gives the following results: I versus II wins II; I versus III wins I; I versus IV wins IV; I versus V wins V; II versus III wins II; II versus IV wins II; II versus V wins II; III versus IV wins IV; III versus V wins V; IV versus V wins IV. The resulting social preference ordering (indicated by SP) is the following: II SP IV SP V SP I SP III, and the preferred alternative is now II.

The example has been chosen to demonstrate that alternative rules for aggregating surpluses may give different results. Surely, it is possible to construct examples in which the above-considered rules of aggregation give the same preferred alternative. More importantly, given the well-known shortcomings of the other two aggregation rules considered, it may be possible to construct examples in which majority voting gives rise to a cycle if preferences are double-peaked for some agent or in which the sum of equivalent variations is not permitted because preferences are not quasi-homothetic or prices differ across consumers.³ In these cases, aggregation by the Borda rule might be a viable alternative.

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³ With reference to the conditions of the Blackorby and Donaldson paper.