

Esercizi proposti - SISTEMI
Matematica Generale II (corso serale)

Sistemi lineari

Risolvere i seguenti sistemi lineari:

1.
$$\begin{cases} 2x + y = 3 \\ 3x + 2z = -1 \\ x - y + 2z = -4 \\ 2y + z = 0 \end{cases} \quad [x = 1, y = 1, z = -2]$$
2.
$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ 3x_1 - x_2 + 3x_3 - x_4 - x_5 = 2 \\ 3x_1 - 5x_2 + 3x_3 - 5x_4 - 5x_5 = 1 \\ -x_1 + 3x_2 - x_3 + 3x_4 + 3x_5 = 1 \end{cases} \quad [\text{incompatibile}]$$
3.
$$\begin{pmatrix} 1 & 9 & 13 & 2 \\ 0 & 2 & 3 & 0 \\ 1 & 5 & 7 & 2 \\ 1 & 3 & 4 & 2 \end{pmatrix} \underline{x} = \begin{pmatrix} 17 \\ 4 \\ 9 \\ 5 \end{pmatrix} \quad [x_1 = \frac{x_3 - 4x_4 - 2}{2}, x_2 = \frac{4 - 3x_3}{2}, \forall x_3, x_4 \in \mathbb{R}]$$
4.
$$\begin{cases} 7x + 6y + 5z = 5 \\ 3x + 4y + 2z = 1 \\ 5x + 6y + 4z = 1 \end{cases} \quad [x = 3, y = -1, z = -2]$$
5.
$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ -1 \end{pmatrix} \quad \left[\underline{x} = \begin{pmatrix} -6 \\ 7 \\ 12 \end{pmatrix} \right]$$
6.
$$\begin{cases} 3x_3 - 2x_4 - 5x_5 = 0 \\ x_1 + 3x_2 + 5x_3 - 4x_4 - 10x_5 = 0 \\ x_1 + 3x_2 + 2x_3 - 2x_4 - 5x_5 = 0 \\ 2x_1 + 6x_2 + 7x_3 - 6x_4 - 15x_5 = 0 \end{cases} \quad \left[\underline{x} = \begin{pmatrix} \frac{-9x_2 + 2x_4 + 5x_5}{3} \\ x_2 \\ \frac{2x_4 + 5x_5}{3} \\ x_4 \\ x_5 \end{pmatrix} \right]$$

$$7. \begin{pmatrix} 1 & 3 & -2 \\ 5 & 15 & -10 \\ 4 & 12 & -7 \\ -2 & -6 & 3 \end{pmatrix} \underline{v} = \underline{0} \left[\underline{v} = \begin{pmatrix} t \\ -\frac{1}{3}t \\ 0 \end{pmatrix} \right]$$

$$8. \begin{cases} \frac{2}{3}x + 2y = 0 \\ x + 3y = 0 \\ 2x - \frac{1}{2}y = 0 \\ 3x + \frac{5}{2}y = 0 \end{cases} [x = 0, y = 0]$$

$$9. \begin{cases} x + 5y + 6z = -3 \\ 7x + y + 2z = 19 \\ 9x + 11y - 10z = 37 \\ 13x - 3y - 2z = 41 \end{cases} [x = 3, y = 0, z = -1]$$

$$10. \begin{cases} x_1 - x_2 + x_3 = 1 \\ 2x_1 - 2x_2 + 2x_3 = 2 \\ -x_1 + x_2 - x_3 = -1 \end{cases} \left[\underline{x} = \begin{pmatrix} 1 + x_2 - x_3 \\ x_2 \\ x_3 \end{pmatrix} \right]$$

$$11. \begin{cases} 2x + y + 2z = 0 \\ 4x + 4y + z = -3 \\ -x - y = 1 \end{cases} \left[\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$12. \begin{cases} -x + 4y + 6z = 4 \\ x - y - 2z = -2 \\ -x + 2y + 3z = 3 \end{cases} \left[\begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} \right]$$

$$13. \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & 1 & 1 \end{pmatrix} \underline{x} = \begin{pmatrix} 3 \\ 1 \\ 10 \end{pmatrix} [impossibile]$$

$$14. \begin{cases} x - y + z - v = 0 \\ x - 2y + 3z - 4v = 0 \\ x - 4y + 7z - 10v = 0 \end{cases} [x = z - 2v, y = 2z - 3v, \forall z, v \in \mathbb{R}]$$

$$15. \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 3 \\ 1 & -3 & 5 \\ 1 & -4 & 7 \end{pmatrix} \underline{x} = \underline{0} \left[\begin{pmatrix} x_3 \\ 2x_3 \\ x_3 \end{pmatrix} \right]$$

Sistemi lineari parametrici

Discutere e risolvere i seguenti sistemi al variare del parametro in $\mathbb{R}$:

1.
$$\begin{cases} (a-1)x + 2ay = -2 \\ 3x - 2y = 8 \end{cases}$$

$$[a \neq 1/4 \Rightarrow \exists! \text{sol.}(x=2, y=-1);$$

$$a = 1/4 \Rightarrow \exists \infty^1 \text{sol.}(x = \frac{2y+8}{3}, \forall y \in \mathbb{R})]$$
2.
$$\begin{cases} x - 2y = -6 \\ 2(k-1)x + y = 4k \\ 2(2-k)x + ky = k \end{cases}$$

$$[k \neq 8 \wedge k \neq 3/4 \Rightarrow \text{incompatibile};$$

$$k = 8 \Rightarrow \exists! \text{sol.}(2, 4)$$

$$k = 3/4 \Rightarrow \exists! \text{sol.}(-12/23, 63/23)]$$
3.
$$\begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix} \underline{x} = \begin{pmatrix} 1 \\ b \end{pmatrix}$$

$$[a = b = 1 \Rightarrow \exists \infty^1 \text{sol.}(x, 1-x);$$

$$a = b = -1 \Rightarrow \exists \infty^1 \text{sol.}(x, -1-x);$$

$$a = b \neq \pm 1 \Rightarrow \text{incompatibile};$$

$$a \neq b \Rightarrow \exists! \text{sol.}(x = (1-b^2)/(a-b), y = (ab-1)/(a-b))]$$
4.
$$\begin{cases} (a-2)x + ay = -a \\ (2-a)x + (2+a)y = -a^2 \end{cases}$$

$$[a = 2 \Rightarrow \exists \infty^1 \text{sol.}(x=t, y=-1);$$

$$a = -1 \Rightarrow \exists \infty^1 \text{sol.}(x=t, y=-3t-1);$$

$$a \neq 2 \wedge a \neq -1 \Rightarrow \exists! \text{sol.}(x = a/2, y = -a/2)]$$
5.
$$\begin{pmatrix} k & 1 & 3 \\ 1 & k & 3 \\ 1 & 3 & k \end{pmatrix} \underline{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$[k = 1 \Rightarrow \exists \infty^1 \text{sol.}(t, (1-t)/4, (1-t)/4);$$

$$k = 3 \Rightarrow \exists \infty^1 \text{sol.}(t, t, (1-4t)/3);$$

$$k = -4 \Rightarrow \text{impossibile};$$

$$k \neq \{1, 3, -4\} \Rightarrow \exists! sol.(x_1 = x_2 = x_3 = 1/(a+4))]$$

$$6. \begin{cases} ax + 3ay - z = -1 \\ (a+1)x + 2y + z = -1 \\ x - ay + 2z = 0 \end{cases}$$